New Approximation Method for Stress Constraints in Structural Synthesis

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A new approximation method for dealing with stress constraints in structural synthesis is presented. In previous methods, it has been common to create an approximation of the actual constraints with respect to the design variables and then use this with optimization to produce a new candidate design. In the present method, the finite-element nodal forces are, instead, approximated, and these are used to create an explicit, but often nonlinear, approximation to the original problem. The principal motivation is to create the best approximation possible, in order to reduce the number of detailed finite-element analyses needed to reach the optimum. The method is shown to be quite simple, while providing significant efficiency gains in the overall optimization task. Also, the method produces such high-quality approximations to the original problem that the need for move limits during the approximate optimization stage is greatly reduced. Examples are offered and compared with published results to demonstrate the efficiency and reliability of the proposed method. It is concluded that there are considerable gains yet to be achieved through continued careful investigation of approximation techniques.

Introduction

ALTHOUGH a wide variety of design objectives and constraints can be included in the structural synthesis problem, virtually all structures must support a set of defined loads without failing due to members being overstressed. Thus, stress constraints are the most common and perhaps the most fundamental constraints imposed on structural design.

Structural optimization methods have evolved dramatically in recent years, owing in large part to our present ability to create high-quality approximations to the structural responses. Using these approximations, the optimization can proceed without repeated and costly large-scale finite-element analyses. The basic approach is to first perform a detailed analysis of the initial proposed design and evaluate all constraints. Then, a sensitivity analysis is performed with respect to the design variables for all critical and near-critical constraints. Using this sensitivity information, an approximate problem is generated, and this is used for optimization. This is usually an explicit problem that can be solved efficiently by a variety of optimization methods. Move limits usually are needed to insure the reliability of the approximation. Once the approximation optimization phase is complete, the new proposed design is analyzed, and the process is repeated until it converges to an acceptable solution.

The key issue is the form of the approximation. Although mixed methods exist, the basic concept of present approximation techniques can be understood by recognizing that static structural response is approximately proportional to the reciprocal of the member sizing variables. Therefore, linearizing with respect to these reciprocal variables offers a high-quality approximation to the original problem. This approximation has been shown to be exact for statically determinate structures and a good approximation for indeterminate structures, where the sizing variables are bar cross sections and membrane thicknesses. When frame structures are considered, the intermediate variables may be taken as the reciprocals of the section properties.

The usual approach is to linearize the actual constraints with respect to these reciprocal variables and then solve this linearized problem.[1] In this reciprocal space the objective becomes nonlinear, but is explicit, and is easily evaluated along with its derivatives. This general approach appears to be quite efficient for displacement constraints and has usually been considered efficient for stress constraints. However, it has been found that an alternative approximation may be preferable for stress constraints, and that is the subject of this paper.

In addition to linearization with respect to reciprocal variables, other work has been directed at a mixed approach of using direct and reciprocal variables,[2,3] The motivation has been to create a convex, separable approximation that is conservative relative to direct linearization and that is well suited for solution by dual methods. Also, these convex linearizations appear to reduce the dependency on direct move limits during the approximate optimization phase.

The motivation for the present work has been somewhat different. Here, we wish to create the highest quality approximation possible, without regard to whether that approximation is convex or has other special properties. Indeed, we argue that if the original problem is nonconvex, then so should be the approximation.

Here, the linearization is carried out on the member forces instead of on the actual stress constraints themselves. The actual sensitivity variables can be the reciprocals of the member section properties. However, the choice of direct vs reciprocal variables becomes far less important than before.

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Problem Formulation

The general optimization problem to be solved is: find the set of design variables X that will

$$\text{Minimize } F(X) \tag{1}$$

Subject to:

$$g_j(X) \leq 0, \qquad j = 1,m \tag{2}$$

$$X_i^\ell \leq X_i \leq X_i^u, \qquad i = 1,n \tag{3}$$

where $F(X)$ and $g_j(X)$ are the objective function and constraints, respectively, and X is the vector of design variables. The total number of constraints is m, and X_i^ℓ and X_i^u are lower and upper bounds on the n design variables, respectively. Here, the objective function is taken to be the structural weight. Constraints considered here include stress, local buckling, and displacement limits. For example, the stress constraints are typically of the form

$$\sigma_{ij}/\bar{\sigma} - 1 \leq 0 \tag{4}$$

where $\bar{\sigma}$ is the allowable stress, i is the load condition, and j is the stress calculation point.

The problem defined by Eqs. (1–3) is a general nonlinear programming task, and a variety of methods and software are available for its solution. In general, to reach an optimum design, the objective and constraint functions and their gradients with respect to the design variables must be evaluated dozens and sometimes hundreds of times. In structural synthesis, this implies a large number of detailed finite-element analyses, which would be prohibitive for practical design. Therefore, approximation techniques are resorted to, whereby the structural responses are approximated using some carefully chosen intermediate variables. From this, an approximate problem is created and solved using nonlinear programming methods. This process is repeated until it converges to an acceptable optimum. The key issue then is the creation of a very high-quality approximation so that the number of detailed finite-element analyses can be as few as possible, and so the approximate optimization phase is not strongly dependent on move limits.

Approximate Problem Formulation

Consider a simple stress constraint for a bar element of the form of Eq. (4). For member i in terms of the member force F_i and the cross-sectional area A_i, the constraint becomes

$$g = F_i/[A_i\bar{\sigma}] - 1 \leq 0 \tag{5}$$

Now, in reciprocal space, letting $X_i = 1/A_i$, the constraint becomes

$$g = F_iX_i/\bar{\sigma} - 1 \leq 0 \tag{6}$$

and the linear approximation of the stress in terms of all design variables X gives

$$g = [\sigma_i^0 + \nabla\sigma_i(X^0) \cdot (X - X^0)]/\bar{\sigma} - 1 \leq 0 \tag{7}$$

where ∇ is the gradient operator.

This linearization includes both the relationship of the force in the member and the member cross-sectional area to the stress and is precise, assuming F_i is constant (which is true for statically determinate structures). However, in general, F_i is not constant, and so a more precise linearization would be

$$g = [F^0 + \nabla F_i(X^0) \cdot (X - X^0)]/[A_i\bar{\sigma}] - 1 \leq 0 \tag{8}$$

which is, in fact, a higher-order approximation to the actual constraint. This is clear from the observation that for a statically determinate structure, the force is constant, and the gradient of the force with respect to the design variables is zero. Thus, only the first term in Eq. (8) is nonzero, whereas both terms are nonzero in Eq. (7). The linearization of F_i with respect to X is indeed simpler to calculate than the linearization of σ. Since the member force is now approximated instead of the stress, the use of reciprocal variables for stress constraints may no longer be the best. Thus, for bar elements, the design variables may be chosen as the direct variables $X_i = A_i$, so that Eq. (8) becomes a linearization in direct space.

Also, although Eq. (8) is nonlinear in A_i, a linear form is easily obtained by multiplying by A_i to give

$$g = [F_i^0 + \nabla F_i(X^0) \cdot (X - X^0)]/\bar{\sigma} - A_i \leq 0 \tag{9}$$

In the case of frame elements, the linearization of forces is created with respect to the member section properties. The nonlinear but explicit constraints are then calculated directly from this. This is a departure from previous methods where the stresses are approximated in terms of the section properties.[4,5]

Although this change in form of the linearized problem is almost trivial, its effect on the optimization task is quite dramatic, as seen from the numerical examples to follow.

Gradient Computations

Calculation of the necessary gradient information follows the usual procedures, except that here the gradients of the element and forces are calculated rather than the gradients of the stresses themselves. For example, using the direct method, the gradient of the joint displacements is first calculated from the familiar relationship:

$$\frac{\partial \boldsymbol{U}}{\partial Y_i} = K^{-1}\left[\frac{\partial \boldsymbol{P}}{\partial Y_i} - \frac{\partial K}{\partial Y_i}\boldsymbol{U}\right] \tag{10}$$

where $\boldsymbol{U}$ is the vector of joint displacements for this loading condition, $\boldsymbol{P}$ is the load vector, and K is the master stiffness matrix; K^{-1} is assumed to be available from the analysis as a decomposed stiffness matrix. The sensitivity variable Y_i is taken here to be the member cross-sectional area for truss elements and the member section property (of which there are six in general) for frame elements.

Now the gradients of the nodal forces in elements are easily calculated from the element force displacement relationship $k_j\boldsymbol{u}_j = \boldsymbol{p}_j$ as

$$\frac{\partial \boldsymbol{p}_j}{\partial Y_i} = \frac{\partial k_j}{\partial Y_i}\boldsymbol{u}_j + k_j\frac{\partial \boldsymbol{u}_j}{\partial Y_i} \tag{11}$$

where $\partial \boldsymbol{u}_j/\partial Y_i$ are the gradients of the joint displacements associated with this element and are recovered from Eq. (10). Therefore, all information needed in Eq. (11) is available from the creation and solution of Eq. (10), and the stress-displacement relationship that is normally used for calculating the gradients of element stresses is not needed at this point.

Implementation

Implementation of the present method follows closely the usual approach for using approximation techniques. The method has been tested for both truss and frame structures. For truss structures, the design variables are taken to be the member cross-sectional areas. For frame structures, the design variables are taken to be the physical member cross-section dimensions. However, the approximation of member end forces is carried out with respect to the member section properties (areas and moments of inertia). Then, when the optimization code requests constraint values, the member section properties are calculated as explicit but nonlinear functions of the design variables. The element end forces are then calculated from the approximation, and finally the stresses are recovered using the element forces, the section properties, and the design variables themselves. Thus, although the approximation of end forces is linear, the actual approximate constraint values can be highly nonlinear, coupled functions of the design variables.

For displacement constraints, truss problems are solved by approximating the displacements with respect to the reciprocal of the member areas in the usual fashion. For frame elements, the displacements are approximated with respect to the reciprocal of the member section properties.

The overall program flow is:

1) Input all analysis and design information.
2) Perform a detailed finite-element analysis.
3) Evaluate all constraints.
4) Delete from consideration those constraints that are not critical or potentially critical during this design cycle. For example, delete all $g_j \leq -0.5$.
5) Calculate the gradients of the joint displacements and element end forces associated with all retained constraints.
6) Create and solve the approximate optimization problem and update the analysis model.
7) Perform a detailed finite-element analysis.
8) Evaluate all constraints.
9) Check for convergence to the optimum. If satisfied, terminate. Otherwise, repeat from step 4.

During the approximate optimization process, numerous candidate designs are created, and the objective and constraint functions must be evaluated for each of these. The program flow to do this is:

1) Output a proposed design X from the optimizer.
2) Calculate the section properties as functions of the design variables.
3) Evaluate the member forces using the approximations.
4) Using the section properties, member forces, and actual member dimensions, calculate the needed stresses.
5) Create the actual constraint functions.
6) Return to the optimizer.

When gradient information is needed by the optimization program, this can be calculated by chain rule differentiation using the available information. In the present study this was not done. Instead, the gradient information for the approximate optimization problem was calculated by first forward finite difference. In practice, this still may be acceptable since the approximate function evaluations are quite cheap compared to a full finite-element analysis. Thus, the approximate optimization uses a relatively small part of the total computational resources.

If there is a compelling reason to create a special approximation for use in optimization, the present approximation can be further approximated leading to an overall multilevel approximation scheme. Indeed, this is exactly what is done when using approximation methods built into a general-purpose optimization program such as the Automated Design Synthesis program, ADS,[6] which was used in this study.

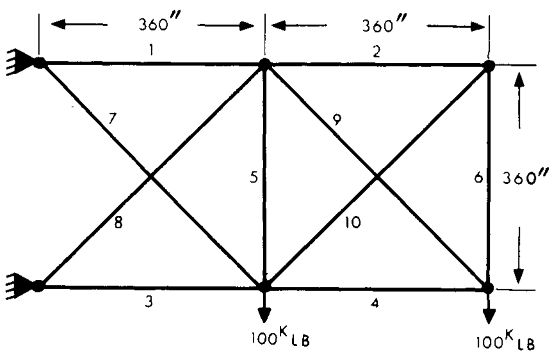

Fig. 1 Ten-bar truss.

Numerical Results

Here, four examples are offered to demonstrate the efficiency and reliability of the method. In the results presented here, one iteration is taken to be one cycle through the program consisting of analysis, calculation of gradients, and creation and solution of the approximate optimization problem. The number of analyses listed in the tables includes the analysis of the initial design.

Case 1: Ten-Bar Truss

Figure 1 shows the ten-bar truss that is commonly used to demonstrate optimization methods. The structure is subjected to one load condition as shown, and the member cross-sectional areas are treated as design variables. The material is aluminum with $E = 10^7$ psi and a specific weight of 0.1 lbm/in.[3]. The allowable stress in each member is $\pm 25{,}000$ psi, except member 9 where the allowable stress is $\pm 50{,}000$ psi. In each approximate optimization, move limits of 90% were used, relative to the design at the beginning of that iteration. For members with small cross-sectional areas, move limits were always taken to be at least ± 1.0 in.^{2}.

Two cases were considered here. In case 1-A, only stress constraints were included. In case 1-B, displacement limits of ± 2.0 in. were imposed in the vertical direction at each joint.

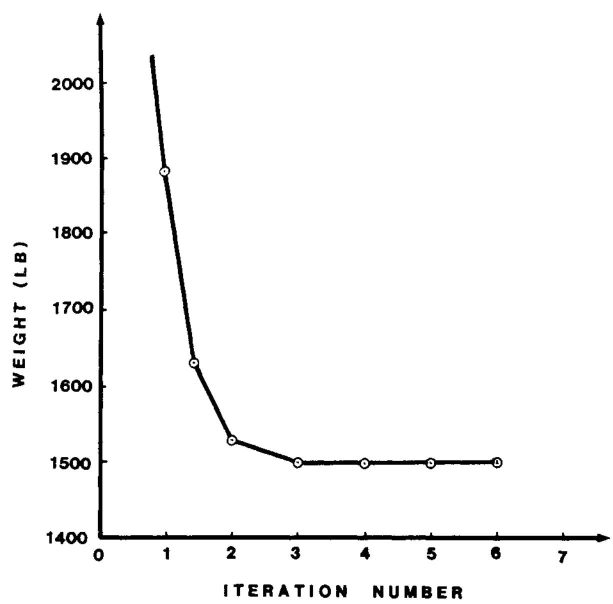

Fig. 2 Iteration history, case 1-A.

Table 1 Results for 10-bar truss

	Member areas				
Member	Initial area	Case 1-A area	Ref. 1 area	Case 1-B area	Ref. 1 area
1	20.0	7.90	7.90	30.52	30.67
2	20.0	0.10	0.10	0.10	0.10
3	20.0	8.10	8.10	23.20	23.76
4	20.0	3.90	3.90	15.22	14.59
5	20.0	0.10	0.10	0.10	0.10
6	20.0	0.10	0.10	0.53	0.10
7	20.0	5.80	5.80	7.46	8.58
8	20.0	5.51	5.52	21.04	21.07
9	20.0	3.67	3.68	21.53	20.96
10	20.0	0.14	0.14	0.10	0.10
Mass	8392.9	1497.4	1497.6	5060.3	5076.9
Number of analyses		7	16	14	13

The iteration history is shown in Fig. 2 for case 1-A and in Fig. 3 for case 1-B. In each case, the design did not change in the last three iterations. The results are given in tabular form in Table 1, together with the results from Ref. 1. It is seen that for stress constraints only, the present method is particularly efficient, whereas the methods are comparable when displacement constraints are added. This is because case 1-B is predominantly controlled by displacement limits, and the two methods use the same approximation for these.

Case 2: Double-Layer Grid

A double-layer grid with a span of 21 m is shown in Fig. 4. This example is taken from Ref. 7. Additional details of the structure are given in Ref. 8. The grid is simply supported at every other boundary joint of the bottom layer. The structure supports a uniformly distributed load on the top layer of 155.5 kg/m^2 that is treated as concentrated vertical loads at the joints. The structure is treated as a truss, and the material properties are $E = 2.1 \times 10^6$ kg/cm^2 with a material density of 0.008 kg/cm^3. Member areas are linked to maintain symmetry about the four lines of symmetry in the plane of the grid. Thus, the problem has 47 independent design variables. The initial areas are taken as 20 cm^2 with a lower bound of 0.1 cm^2.

Three cases are considered. In case 2-A, stress and Euler buckling constraints are considered, whereas in cases 2-B and 2-C displacement limits are also imposed. The allowable stress is taken as -1000 kg/cm$^2 \leq \sigma i \leq 1400$ kg/cm^2, $i = 1,47$, where i is the element number. The members are taken to be tubular with a mean diameter to wall thickness ratio of 10. The members are also constrained by the Euler buckling limit of

$$\sigma i \geq \sigma_{bi} = -10.1\pi E A_i / 8 L_i^2, \qquad i = 1,47$$

where A_i and L_i are the area and length, respectively, of member i. For case 2-B, displacement constraints of ± 2.5 cm were imposed at each joint in each coordinate direction, whereas in case 2-C this limit is reduced to ± 1.5 cm.

The iteration history for each case is shown in Fig. 5, and the numerical results are given in Table 2. In all cases, move limits of 90% were used. For case 2-C, the displacement constraints dominate the design, and convergence is somewhat slower. In each case, the approximation was found to be unconservative, leading to constraint violations in the early design iterations, but which were overcome at the optimum.

Case 3: Grillage

Figure 6 shows a grillage that has been used elsewhere as a design example. In this example, the members are assumed to be frame elements with cross-sectionals as shown in the figure. Each member is defined by four design variables as shown, and the structure is required to be symmetric, leading to a total of

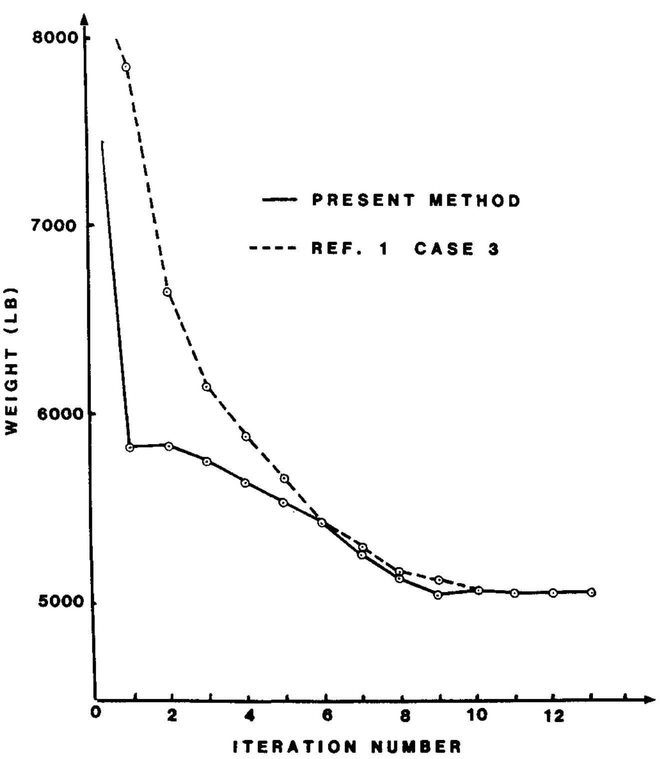

Fig. 3 Iteration history, case 1-B

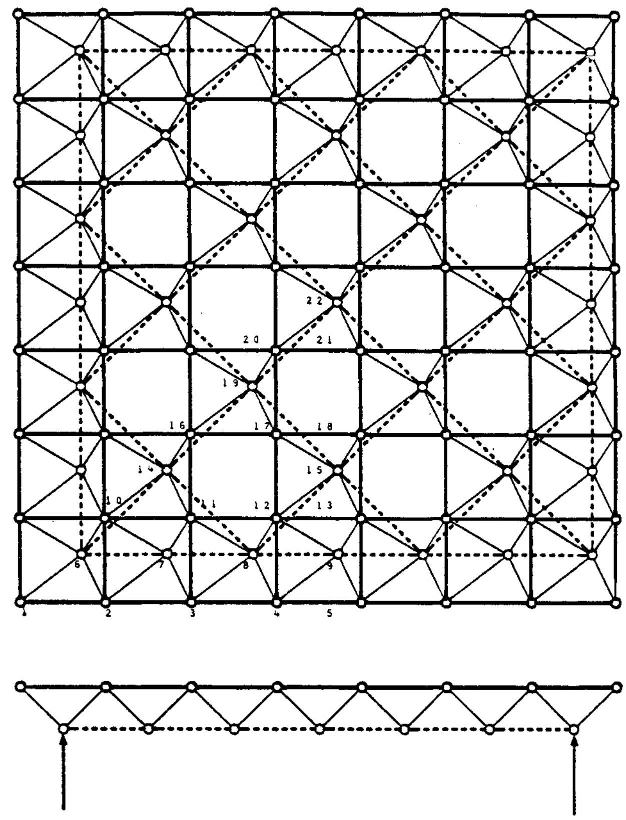

Fig. 4 Double-layer grid.

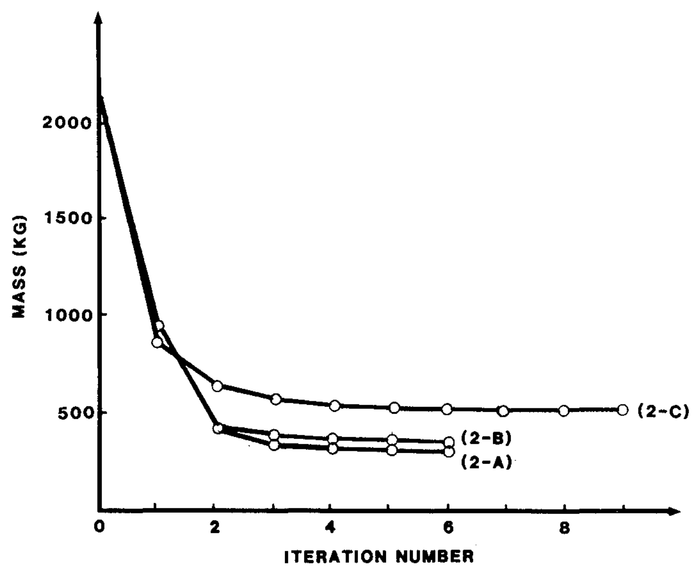

Fig. 5 Iteration history, case 2.

Table 2 Final designs for double-layer grid

Members connecting joints	Areas (cm^2) Case 2-A	Case 2-B	Case 2-C	Members connecting joints	Areas (cm^2) Case 2-A	Case 2-B	Case 2-C
1–2	1.94	1.94	1.94	19–22	8.89	8.53	9.9
2–3	0.18	0.23	0.2	6–2	0.76	0.80	0.75
3–4	2.75	2.76	2.75	10–7	0.35	0.36	0.68
4–5	0.1[a]	0.25	0.16	7–3	1.1	1.12	1.11
10–11	0.14	0.1[a]	0.1[a]	14–11	3.04	3.01	2.99
11–12	5.71	5.71	5.76	11–8	2.86	2.95	5.46
12–13	1.89	2.57	6.19	8–4	0.86	0.86	0.86
16–17	2.33	4.15	8.49	12–9	0.1[a]	0.1[a]	0.1[a]
17–18	0.28	0.1[a]	0.1[a]	19–17	3.22	3.3	5.11
20–21	6.6	16.73	32.11	17–15	2.64	4.42	8.83
2–10	2.75	2.75	2.75	2–7	0.1[a]	0.13	0.11
3–11	2.75	2.75	2.75	7–11	1.88	1.8	1.88
11–16	1.61	2.11	3.93	3–8	0.97	0.97	0.97
4–12	2.75	2.75	2.75	8–12	4.37	5.91	11.25
12–17	3.55	5.73	11.06	4–9	0.1[a]	0.1[a]	0.1[a]
17–20	5.6	10.13	19.55	12–15	5.47	5.53	8.78
6–7	2.57	2.5	2.62	1–6	0.43	0.43	0.43
7–8	0.1[a]	0.1[a]	0.1[a]	6–10	1.11	1.09	1.05
8–9	5.22	5.30	4.54	10–14	0.3	0.11	0.1[a]
14–8	9.43	9.42	8.77	14–16	1.63	1.91	3.87
19–15	13.23	13.34	19.36	16–19	0.15	0.2	0.21
8–15	6.66	6.68	7.55	19–20	1.73	4.51	8.72
6–14	0.13	0.11	0.47	20–22	0.1[a]	0.1[a]	0.1[a]
14–19	6.21	5.91	5.66				
Weight (kg)	327.26	373.32	520.36				
No. of analyses	7	7	10				

[a]Sizing variable at lower bound.

Table 3 Initial design and side constraints for grillage

Members	Sizing variables	Initial value (cm)	Lower bound (cm)	Upper bound (cm)
	B	30.48	2.54	48.26
	t_b	2.41	0.114	2.54
All members	H	38.10	2.54	50.8
	t_h	2.03	0.127	2.41

16 independent design variables. The structure is required to support a uniform load. Two cases are presented here. In case 3-A, stress limits are imposed at eight points, and local buckling constraints are imposed on each of the four walls at each end of each element. Case 3-B is the same, with the addition of displacement constraints. From symmetry, only one-fourth of the structure was modeled. This example is also used in Ref. 9, and the details of materials, loading, and constraints are given there. The initial design and the bounds on the design variables are given in Table 3. The iteration history is shown in Fig. 7 for each case, and the results are tabulated in Table 4. In each case, move limits of 90% were used throughout the optimization process. The results for case 3-B were also presented in Ref. 9 where the best solution required 11 analyses. In Ref. 7, additional cases are offered where single and multiple frequency constraints are imposed in addition to those considered here.

Case 4: Portal Frame

As a final example, the portal frame shown in Fig. 8 was designed for minimum weight. Two load conditions are considered, as shown in the figure together with the material properties and stress limits. The details of this example are given in Ref. 10. Again, two cases are considered. In case 4-A, the initial design is feasible, whereas in case 4-B the initial design is infeasible. The initial and final designs are given in Table 5, together with those from Ref. 4 for case 4-A. The iteration history for each case is shown in Fig. 9. The method presented in Ref. 4 required a total of 11 analyses for its solution.

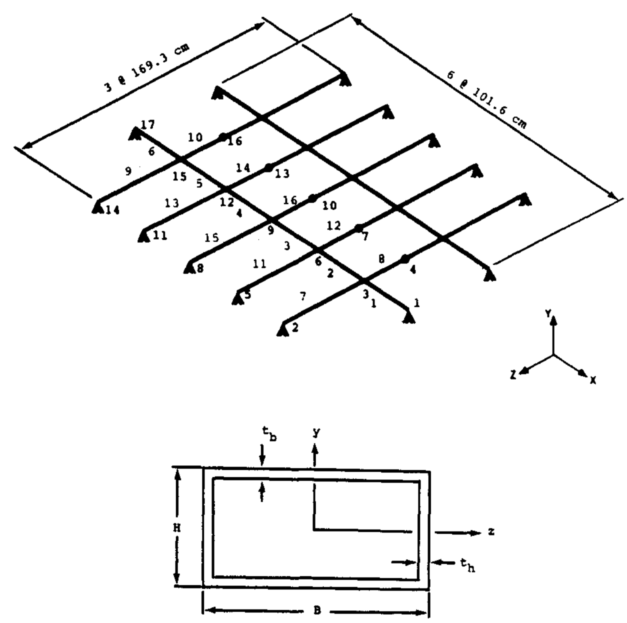

Fig. 6 Grillage.

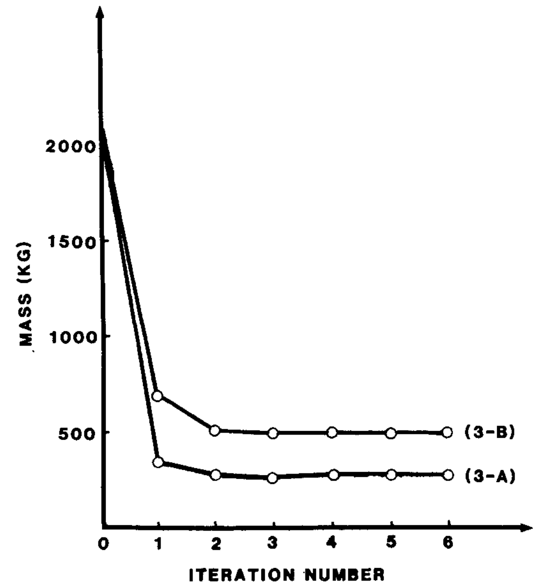

Fig. 7 Iteration history, case 3.

Table 4 Final designs for grillage

Linking group	Members	Size var.	Case 3-A (cm)	Case 3-B (cm)
		B	12.93	24.38
		t_b	0.35	0.24
1	1–6	H	34.96	50.80[a]
		t_h	0.21	0.24
		B	13.38	10.72
		t_b	0.18	0.14
2	7–10	H	22.15	22.49
		t_h	0.14	0.14
		B	32.40	48.26[a]
		t_b	0.57	0.81
3	11–14	H	50.80[a]	50.80[a]
		t_h	0.32	0.25
		B	23.08	48.26[a]
		t_b	0.42	1.39
4	15–16	H	46.42	50.80[a]
		t_h	0.27	0.29
Weight (kg)			275.64	498.56
Number of analyses			7	7

[a]Sizing variable at upper bound.

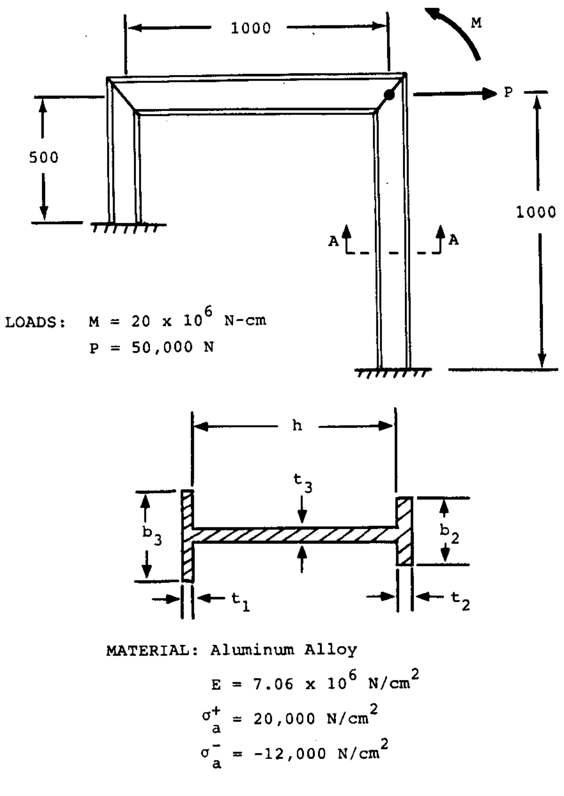

Fig. 8 Portal frame.

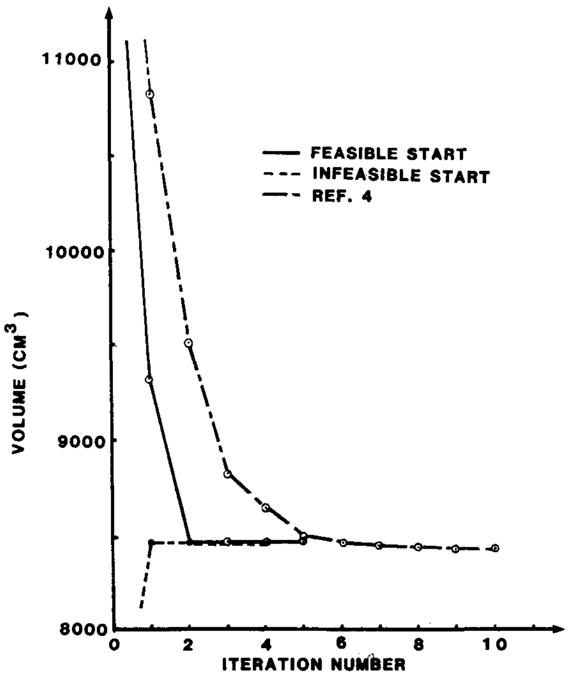

Fig. 9 Iteration history, case 4.

Discussion

A new linearized form of the approximate optimization task is offered, relative to stress constraints. The principal motivation has been to create the highest quality approximation possible. Once this is done, the method for solving the actual approximate optimization problem becomes one of choice. In the examples presented here, the approximate problem was solved as a primal optimization task. The essential ingredient is that the number of expensive detailed structural analyses is dramatically reduced. Furthermore, this approach does not preclude using whatever linearization is best for other constraints. The necessary information is available from the original sensitivity analysis and only needs to be manipulated to obtain the highest quality approximation possible.

For truss structures, the element end forces are approximated with respect to the cross-sectional areas, whereas for frame elements the end forces are approximated with respect to the section properties (which are equivalent for trusses since the only section property is the area). This information then is appropriately manipulated relative to the actual design vari-

Table 5 Results for portal frame[a]

Member	Variable	Case 4-A Initial	Case 4-A Optimum	Ref. 4	Case 4-B Initial	Case 4-B Optimum
	B_1	30.0	9.07	11.26	10.0	5.00
	t_1	1.0	0.52	0.41	0.5	1.03
1	H	52.0	79.13	78.21	20.0	78.87
	t_3	1.0	0.52	0.52	0.5	0.52
	B_3	30.0	10.00	10.17	10.0	10.0
	t_2	1.0	0.48	0.46	0.5	0.50
	B_1	30.0	9.34	11.69	10.0	5.00
	t_1	1.0	0.56	0.42	0.5	1.10
	H	52.0	100.00	99.47	20.0	100.00
2	t_3	1.0	0.43	0.44	0.5	0.43
	B_2	30.0	10.36	10.94	10.0	10.0
	t_2	1.0	0.50	0.45	0.5	0.52
	B_1	30.0	5.00	5.00	10.0	5.00
	t_1	1.0	0.17	0.14	0.5	0.17
	H	52.0	25.00	25.00	20.0	25.00
3	t_3	1.0	0.10	0.10	0.5	0.10
	B_2	30.0	10.00	10.00	10.0	10.00
	t_2	1.0	0.25	0.28	0.5	0.25
Volume		275,000	84,367	84,058	48,750	84,282

[a]Ref. 4 had 0.3% constraint violation. Present method had a maximum of 0.1% constraint violation.

ables to produce a high-quality approximate optimization problem. This has the particular advantage for frame structures that the actual design variables are the physical variables that the engineer is accustomed to working with. The method has been found to be insensitive to move limits and usually converges to a near optimum in only two or three design iterations.

It is concluded that there are considerable gains yet to be achieved through careful investigation of approximation techniques.

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